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Non-relativistic superbranes

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Abstract: Subtleties arising in the non-relativistic limit of relativistic branes are resolved, and a reparametrization-invariant and kappa-symmetric non-relativistic super p-brane action is obtained as a limit of the action for a relativistic super p-brane in a Minkowski vacuum. We give explicit results for the D0-brane, which provides a realization of the super-Bargmann algebra, the IIA superstring and the 11-dimensional supermembrane.

Keywords: Superstrings and Heterotic Strings, p-branes, Brane Dynamics in Gauge Theories, Space-Time Symmetries.

Contents

1. Introduction

It is often useful to consider special limits of physical theories and the non-relativistic limit of relativistic theories is a good example. Non-relativistic particle mechanics is of course a venerable subject and so, in a sense, is the non-relativistic mechanics of branes; we have in mind the textbook studies of waves on strings and membranes, but since these involve some material medium their dynamics is not usefully thought of as the non-relativistic limit of some relativistic system. Alternatively, one can start with a relativistic p-brane and take its non-relativistic limit. This is standard for $p = 0$, although there are some well-known subtleties, but a new difficulty occurs for $p > 0$. In these cases, each p-volume element, or 'particle', of a closed p-brane has its 'anti-particle' elsewhere on the brane. As the characteristic feature of a non-relativistic theory is the absence of anti-particles, any non-relativistic limit of a p-brane must decouple 'opposite' p-volume elements. One can achieve this decoupling by rescaling the spacetime coordinates in order to focus on a small region of the brane and then taking a limit that decouples it from other regions. Let us illustrate this procedure with the action

$$S = -T_p \int d^{p+1}\xi \sqrt{-\det G} \tag{1.1}$$

for a bosonic p-brane in a d-dimensional product spacetime, with metric

$$ds^2 = \omega^2 \eta_{\mu\nu} dx^\mu dx^\nu + G_{ab}(X) dX^a dX^b\,, \tag{1.2}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$ is the Minkowski $(p+1)$-metric (in cartesian coordinates), and G_{ab} is a metric on some $(d-p-1)$-dimensional riemannian manifold $\mathcal{M}$. We have rescaled the coordinates of the Minkowski factor by introducing an arbitrary dimensionless constant ω. The induced worldvolume metric is

$$G_{ij} = \omega^2 g_{ij} + \partial_i X^a \partial_j X^b G_{ab}\,, \tag{1.3}$$

where

$$g_{ij} = \partial_i x^\mu \partial_j x^\nu \eta_{\mu\nu} \tag{1.4}$$

is the Minkowski metric in the arbitrary coordinates ξ^i. It follows that

$$\sqrt{-\det G} = \omega^{p+1}\sqrt{-\det g}\left[1 + \frac{1}{2\omega^2} g^{ij}\partial_i X^a \partial_j X^b G_{ab} + \mathcal{O}\left(\frac{1}{\omega^4}\right)\right]. \tag{1.5}$$

Defining a rescaled p-volume tension T by

$$T_p = \omega^{1-p}\, T\,, \tag{1.6}$$

we find that

$$S = -T\int d^{p+1}\xi\left[\omega^2\,\sqrt{-\det g} + \frac{1}{2}\sqrt{-\det g}\, g^{ij}\partial_i X^a \partial_j X^b G_{ab} + \cdots\right], \tag{1.7}$$

where the omitted terms vanish in the $\omega \to \infty$ limit, and are higher-order in derivatives. In effect, the expansion in inverse powers of ω is an expansion in derivatives, so the leading terms provide an effective low-energy action. This is generally what is meant by the 'field theory limit', which therefore has an interpretation as a non-relativistic limit [1]–[4].

Note now that $\sqrt{-\det g}$ is the jacobian for the change of Minkowski space coordinates from the cartesian coordinates x^μ to the arbitrary coordinates ξ^i, so the first term in (1.7) is the integral over the Minkowski spacetime of a constant proportional to $\omega^2 T$. Alternatively, one may observe that $\sqrt{-\det g}$ is a total derivative with respect to the ξ coordinates. From either perspective, it is clear that this term may be omitted. For $p = 0$ this corresponds to the subtraction of the rest-mass energy from the total energy. For $p = 1$ it corresponds to the subtraction procedure advocated in [2, 3]. Having removed this term we can take the $\omega \to \infty$ limit to arrive at the (still reparametrization invariant) *non-relativistic* p-brane action

$$S = -\frac{1}{2}T\int d^{p+1}\xi\,\sqrt{-\det g}\, g^{ij}\partial_i X^a \partial_j X^b G_{ab}\,. \tag{1.8}$$

For $p = 0$, we can set $T = mc$ and $x^0(\xi^0) = ct(\xi^0)$ to get the standard time-reparametrization invariant action for a non-relativistic particle on $\mathcal{M}$:

$$S_0 = m\int d\xi^0\,\frac{|\dot X|^2}{2\dot t}\,. \tag{1.9}$$

The action (1.8) generalizes this to $p > 0$.

If we choose the Monge gauge

$$\partial_i x^\mu = \delta_i{}^\mu \tag{1.10}$$

to fix the reparametrization invariance of (1.8) then we get the standard Minkowski space sigma-model action

$$S = -\frac{1}{2}T\int d^{p+1}\xi\,\eta^{ij}\partial_i X^a \partial_j X^b G_{ab}(X)\,. \tag{1.11}$$

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Note that disturbances of the sigma-model fields propagate at the speed of light, which we could have set to unity from the beginning. This may seem puzzling, because one is used to thinking of the non-relativistic limit as one in which $c \to \infty$, but c is not dimensionless so, strictly speaking, it makes no sense to think of it as a variable that we can take to infinity [5]. For particles, one can get away with thinking of the non-relativistic limit as one in which $c \to \infty$. However, it is preferable not to think of it this way, and essential not to do so for branes.

The purpose of this paper is to generalize some of the above discussion to super-p-branes. The most general action that we will consider here takes the form

$$S = -T_p \int d^{p+1}\xi\, \sqrt{-\det G} + Q_p \int b\,, \tag{1.12}$$

where the worldvolume metric G is a superspace extension of the induced worldvolume metric and b is a superspace $(p+1)$-form constructed from a super-Poincaré invariant closed $(p+2)$-form $h = db$. For the moment, we allow this 'Wess-Zumino' (WZ) term to have an arbitrary coefficient Q_p, which can be interpreted as the p-brane charge. We will suppose that the superspace background is the Minkowski vacuum of some supergravity theory, and that the action is invariant under the action of the super-Poincaré isometry group of this background. In particular, we assume invariance under the infinitesimal supersymmetry transformation

$$\delta\theta = \epsilon\,, \qquad \delta X^m = -i\bar\epsilon\Gamma^m\theta\,. \tag{1.13}$$

We suppose, for the moment, that θ is a minimal spinor. For simplicity of presentation we also assume that the spacetime dimension d is such that the Dirac matrices Γ^m ($m = 0, 1, \dots, d-1$) can be chosen to be real, in which case θ is real, and

$$\bar\theta = \theta^T\Gamma^0\,. \tag{1.14}$$

In other words, we assume that θ is Majorana (or Majorana-Weyl) and choose a (real) basis of the Dirac matrices for which the charge conjugation matrix C is equal to Γ^0.

Both the induced metric G_{ij} and the superspace $(p+2)$-form h are constructed from the super-translation invariant 1-forms $d\theta$ and

$$\Pi^m = dX^m + i\bar\theta\Gamma^m d\theta\,. \tag{1.15}$$

Specifically,

$$G_{ij} = \Pi_i{}^m \Pi_j{}^m \eta_{mn}\,, \tag{1.16}$$

where $\Pi_i{}^m$ are the components of the worldvolume 1-forms induced by Π^m, and

$$h = -\frac{i}{p!}\, \Pi^{m_1} \wedge \cdots \wedge \Pi^{m_p} d\bar\theta\Gamma_{m_1\dots m_p} d\theta\,. \tag{1.17}$$

We must assume that h does not vanish identically. The requirement that $dh = 0$ is then equivalent to [6]

$$\left(\bar u\Gamma^m u\right)\left(\bar u\Gamma_{mn_1\dots n_p} u\right) \equiv 0 \tag{1.18}$$

for *commuting* minimal spinor u. Given our restriction to spacetime dimensions for which the Dirac matrices are real, this restricts the worldspace dimension to $p = 1, 2, \bmod 4$. This does not include $p = 0$, but we will later consider separately the D0-brane (for which the spinor θ is not minimal).

Suppose now that we rescale the superspace coordinates by setting

$$X^m = (\omega x^\mu, X^a)\,, \qquad \theta = \frac{1}{\sqrt{\omega}}\,\vartheta\,. \tag{1.19}$$

The induced metric is then

$$G_{ij} = \omega^2\left[g_{ij} + \frac{1}{\omega^2}\,\partial_i\mathbf{X}\cdot\partial_j\mathbf{X} + \frac{2}{\omega^2}\,i\bar\vartheta\gamma_{(i}\partial_{j)}\vartheta + \mathcal{O}\left(\frac{1}{\omega^4}\right)\right], \tag{1.20}$$

where

$$\gamma_i = \partial_i x^\mu \Gamma_\mu \tag{1.21}$$

and hence

$$\sqrt{-\det G} = \omega^{p-1}\sqrt{-\det g}\left[\omega^2 + \frac{1}{2}g^{ij}\partial_i\mathbf{X}\cdot\partial_j\mathbf{X} + i\bar\vartheta\gamma^i\partial_i\vartheta + \mathcal{O}\left(\frac{1}{\omega^2}\right)\right], \tag{1.22}$$

where $\gamma^i = g^{ij}\gamma_j$. Similarly,

$$h = -\frac{i}{p!}\,\omega^{p-1}\left[dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_p}d\bar\vartheta\Gamma_{\mu_1\ldots\mu_p}d\vartheta + \mathcal{O}\left(\frac{1}{\omega^2}\right)\right], \tag{1.23}$$

which leads to a $(p+1)$-form b that is the worldvolume Poincaré dual of the WZ scalar density

$$\mathcal{L}_{\rm WZ} = \omega^{p-1}\left[\sqrt{-\det g}\;i(\bar\vartheta_-\gamma^i\partial_i\vartheta_- - \bar\vartheta_+\gamma^i\partial_i\vartheta_+) + \mathcal{O}\left(\frac{1}{\omega^2}\right)\right], \tag{1.24}$$

where $\vartheta_\pm$ are eigenspinors with eigenvalue ± 1 of the matrix

$$\Gamma_* = \frac{1}{(p+1)!}\,\varepsilon^{\mu_1\cdots\mu_{p+1}}\Gamma_{\mu_1\ldots\mu_{p+1}}, \tag{1.25}$$

which satisfies $\Gamma_*^2 = 1$ as a consequence of the previously mentioned restriction to $p = 1, 2$ mod 4. Note that we now have an expansion 'in derivatives' for which a fermion counts as 'half a derivative', as is generally the case for low-energy expansions, so the limit as $\omega\to\infty$ can again be viewed as a 'field theory' limit. If we now define a rescaled tension T and rescaled charge Q by

$$T_p = \omega^{1-p}\,T, \qquad Q_p = \omega^{1-p}\,Q\,, \tag{1.26}$$

and follow the same procedure as before, then we arrive at the free-field theory with lagrangian density

$$\mathcal{L} = -\frac{T}{2}\eta^{ij}\partial_i\mathbf{X}\cdot\partial_j\mathbf{X} - i\,(T+Q)\,\bar\vartheta_+\gamma^i\partial_i\vartheta_+ - i\,(T-Q)\,\bar\vartheta_-\gamma^i\partial_i\vartheta_-\,. \tag{1.27}$$

Note that $T < |Q|$ implies ghosts in the quantum theory, so that T should satisfy the bound $T \geq |Q|$.

So far, our extension of the non-relativistic limit of bosonic p-branes to super-p-branes appears satisfactory, and it is for $T > |Q|$. Before the rescaling of coordinates, the action was invariant under the rigid supersymmetry transformations (1.13). For these transformations to have a sensible $\omega \to \infty$ limit after the rescaling, we must define a rescaled supersymmetry parameter $\varepsilon = \sqrt{\omega}\,\epsilon$. Then, in the $\omega \to \infty$ limit, the only non-zero 'supersymmetry' transformation is

$$\delta\vartheta = \varepsilon\,. \tag{1.28}$$

This is certainly a symmetry of the non-relativistic action, and is all one should expect for $T > |Q|$; it is the linearized remnant of a non-linear realization of spacetime supersymmetry. However, the $T = |Q|$ case is clearly special: in this case we know that the relativistic action has a fermionic gauge invariance, called 'kappa-symmetry', that allows half the fermions to be 'gauged away', so it should not be a surprise that half of the fermions drop out in the non-relativistic limit. However, we also know that when $T = |Q|$ the gauge-fixed relativistic action has a linearly-realized worldvolume supersymmetry [7, 8]. We would expect this feature to survive the non-relativistic limit, and it is obvious that the free field lagrangian (1.27) *is* supersymmetric when $T = |Q|$, because of the match of bose/fermi degrees of freedom implied by the assumed restrictions on (d, p) [9]. However, *this linearly-realized worldvolume supersymmetry of the non-relativistic super-p-brane action appears not to be the limit of any symmetry of the relativistic action.*

The resolution of the puzzle is that precisely when $T = |Q|$, but not otherwise, there is *another* non-relativistic limit, differing from the one just described in the way that the fermionic variables are rescaled. Instead of (1.19) we may set

$$X^m = (\omega x^\mu, X^a)\,, \qquad \theta = \sqrt{\omega}\,\theta_- + \frac{1}{\sqrt{\omega}}\,\theta_+\,, \tag{1.29}$$

where $\theta_\pm$ are ± 1 eigenspinors of Γ_*. Because θ_- is scaled by a *positive* power of ω, the action for general T, Q has no $\omega \to \infty$ limit. However, a cancellation occurs for $T = |Q|$ that makes the limit possible (once a constant 'rest-energy' term has been discarded). Note that the new limit is *not* one in which all fermions count as 'half a derivative'; this is true only of θ_+. However, we will see that kappa-symmetry will allow us to set $\theta_- = 0$, so that the new limit is still a field theory limit in terms of the physical worldvolume fields. For the Minkowski background considered here, both 'old' and 'new' non-relativistic limits yield the same final (free-field) action, *but only in the new limit is the linearly realized supersymmetry of this action obtained as a limit of the symmetries of the relativistic action.*

We will begin with a discussion of the non-relativistic limit of the kappa-symmetric super-p-brane action, along the lines indicated above. We will show that this limit results in a non-relativistic super-p-brane action that is both invariant under a non-relativistic spacetime supersymmetry, and gauge invariant with respect to a non-relativistic kappa-symmetry. This action is given implicitly for general p, in terms of a 'non-relativistic' closed superspace $(p+2)$-form h_{nr}, but we give the explicit result for $p = 2$, which includes the case of the 11-dimensional supermembrane (M2-brane). Moreover, the gauge fixed action is easily found for general (admissible) p, and the result is shown to be a $(p+1)$-dimensional

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super-Poincaré invariant worldvolume field theory. This transmutation of spacetime into worldvolume supersymmetry occurs also for the relativistic super-p-brane, but it is much easier to see in the non-relativistic limit.

We then move to a discussion of the non-relativistic limit of the D0-brane. This is properly thought of as the $p = 0$ subcase of the super D-p-brane rather than as the $p = 0$ subcase of the super-p-brane that we have been describing above, so there are a few differences. However, the basic idea is the same, and this case has the advantage that the explicit gauge-invariant non-relativistic action is simple. For this case, we also consider the effect of the non-relativistic limit on the super-Poincaré symmetry algebra of the D0-brane. One of the subtleties of the non-relativistic limit alluded to earlier is that the symmetry algebra of the non-relativistic bosonic particle is *not* the Galilei algebra obtained by contraction of the Poincaré symmetry algebra of the relativistic particle, but a central extension of it called the Bargmann algebra. The analogous central charges for $p > 0$ were discussed in a recent article [10]. For the D0-brane, however, the super-Bargmann symmetry algebra of the non-relativistic D0-brane *is* the contraction of the super-Poincaré symmetry algebra of the relativistic D0-brane, and we show explicitly how it is realized.

Finally, we consider the IIA superstring, presenting an explicit reparametrization invariant and kappa-symmetric non-relativistic action, and conclude with some comments on future directions

2. The non-relativistic super-p-brane

We now concentrate on the kappa-symmetric super-p-brane action

$$S = -T_p \int d^{p+1}\xi \left[\sqrt{-\det G} - \mathcal{L}_{\mathrm{WZ}}\right], \tag{2.1}$$

where $\mathcal{L}_{\mathrm{WZ}}$ is the Wess-Zumino lagrangian density. This action is invariant under the 'kappa'-gauge transformation

$$\delta\theta = \frac{1}{2}\left(1 - \Gamma_\kappa\right)\kappa(\xi), \qquad \delta X^m = -i\bar{\theta}\Gamma^m\delta\theta\,, \tag{2.2}$$

where [11, 6]

$$\sqrt{-\det G}\;\Gamma_\kappa = \frac{1}{(p+1)!}\,\varepsilon^{i_1\dots i_{p+1}}\Pi_{i_1}^{m_1}\cdots\Pi_{i_{p+1}}^{m_{p+1}}\Gamma_{m_1\dots m_{p+1}}\,. \tag{2.3}$$

This matrix satisfies $\Gamma_\kappa^2 = 1$ (assuming the restrictions spelled out earlier).

To take the non-relativistic limit, we first define the 1-forms

$$e^\mu = dx^\mu + i\overline{\theta}_-\Gamma^\mu d\theta_-\,, \qquad u^a = dx^a + 2i\overline{\theta}_+\Gamma^a d\theta_-\,, \tag{2.4}$$

where (for later convenience) we have introduced the new transverse space coordinate

$$x^a = X^a + i\overline{\theta}_-\Gamma^a\theta_+\,. \tag{2.5}$$

We then define rescaled coordinates as in (1.29), so that

$$\Pi_i^\mu = \omega e_i{}^\mu + \frac{i}{\omega}\bar\theta_+\Gamma^\mu\partial_i\theta_+, \qquad \Pi_i{}^a = u_i{}^a\,. \tag{2.6}$$

This yields

$$G_{ij} = \omega^2\,\hat g_{ij} + 2i\bar\theta_+\hat\gamma_{(i}\partial_{j)}\theta_+ + \mathbf{u}_i\cdot\mathbf{u}_j + \mathcal{O}\left(\frac{1}{\omega^2}\right), \tag{2.7}$$

where[1]

$$\hat g_{ij} = e_i{}^\mu e_j{}^\nu\eta_{\mu\nu}, \qquad \hat\gamma_i = e_i{}^\mu\Gamma_\mu\,. \tag{2.8}$$

Noting that

$$\sqrt{-\det\hat g} = \det e_i{}^\mu =: e \tag{2.9}$$

we now find that

$$\sqrt{-\det G} = \omega^{p-1}\,e\left[\omega^2 + \frac{1}{2}\hat g^{ij}\mathbf{u}_i\cdot\mathbf{u}_j + i\bar\theta_+\hat\gamma^i\partial_i\theta_+ + \mathcal{O}\left(\frac{1}{\omega^2}\right)\right], \tag{2.10}$$

where $\hat g^{ij}$ is the inverse of $\hat g_{ij}$ and $\hat\gamma^i = \hat g^{ij}\hat\gamma_j$.

To complete the expansion of the action in inverse powers of ω, we need the expansion of the superspace $(p+2)$-form h. With the exterior product of forms being understood, this is

$$h = \omega^{p+1}h_0 + \omega^{p-1}\,h_{nr} + \mathcal{O}(\omega^{p-3})\,, \tag{2.11}$$

where

$$h_0 = -\frac{i}{p!}\left(d\bar\theta_-\hat\gamma_{i_1\ldots i_p}d\theta_-\right)d\xi^{i_1}\cdots d\xi^{i_p} \tag{2.12}$$

and

$$\begin{aligned} h_{nr} = -\frac{i}{p!}\Big[&\left(d\bar\theta_+\hat\gamma_{i_1\ldots i_p}d\theta_+\right) + \\ &+ p\left(d\bar\theta_-\hat\gamma_{i_1\ldots i_{p-1}}\Gamma_\mu d\theta_-\right)\left(i\bar\theta_+\Gamma^\mu\partial_{i_p}\theta_+\right) + 2p\left(d\bar\theta_+\hat\gamma_{i_1\ldots i_{p-1}}\Gamma_a d\theta_-\right)u_{i_p}{}^a + \\ &+ \frac{p(p-1)}{2}\left(d\bar\theta_-\hat\gamma_{i_i\ldots i_{p-2}}\Gamma_{ab}d\theta_-\right)u_{i_{p-1}}{}^a u_{i_p}{}^b\Big]d\xi^{i_1}\cdots d\xi^{i_p}\,. \end{aligned} \tag{2.13}$$

The leading term h_0 is not manifestly a closed form because $d\hat\gamma^i$ is non-zero. However, one can show by means of the identity (1.18) and a lengthy algebraic manipulation (which we omit) that $h_0 = db_0$, where

$$b_0 = \left(d\xi^0\ldots d\xi^p\right)\left[\sqrt{-\hat g} - \sqrt{-g}\right]. \tag{2.14}$$

Similarly, it follows from the closure of h that h_{nr} is closed, so we can write (at least locally)

$$h_{nr} = db_{nr} \tag{2.15}$$

[1]The hats are to remind us that these quantities depend on θ_-.

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for some $(p+1)$-form b_{nr}; its explicit form is not easy to determine for general p, so we will limit ourselves here to giving it for $p=2$,

$$\begin{aligned} b_{nr} = & -\frac{1}{2}\Big\{ K^{+}_{\mu\nu}\left[e^{\mu}e^{\nu} - K^{\mu}_{-}e^{\nu} + \frac{1}{3}K^{\mu}_{-}K^{\nu}_{-}\right] + K^{-}_{\mu\nu}K^{\mu}_{+}\left[e^{\nu} - \frac{1}{3}K^{\nu}_{-}\right] + \\ & + L_{\mu b}\left[2e^{\mu}u^{b} - e^{\mu}L^{b} - K^{\mu}_{-}u^{b} + \frac{2}{3}K^{\mu}_{-}L^{b}\right] + \\ & + K^{-}_{ab}\left[u^{a}u^{b} - L^{a}u^{b} + \frac{1}{3}L^{a}L^{b}\right]\Big\}, \end{aligned} \tag{2.16}$$

where

$$\begin{aligned} K^{\pm}_{\mu\nu} &= i\bar{\theta}_{\pm}\Gamma_{\mu\nu}d\theta_{\pm}\,, \qquad K^{\mu}_{\pm} = i\bar{\theta}_{\pm}\Gamma^{\mu}d\theta_{\pm}\,, \qquad K^{-}_{ab} = i\bar{\theta}_{-}\Gamma_{ab}d\theta_{-}\,, \\ L_{\mu a} &= i\bar{\theta}_{+}\Gamma_{\mu}\Gamma_{a}d\theta_{-} + i\bar{\theta}_{-}\Gamma_{\mu}\Gamma_{a}d\theta_{+}\,, \qquad L^{a} = i\bar{\theta}_{+}\Gamma^{a}d\theta_{-} + i\bar{\theta}_{-}\Gamma^{a}d\theta_{+}\,. \end{aligned} \tag{2.17}$$

This case is of particular interest as it includes the 11-dimensional supermembrane [6].

We are now in a position to compute the non-relativistic limit of the action. In terms of the rescaled tension T, the total lagrangian density is

$$-T_p\sqrt{-\det G} + T_p\mathcal{L}_{\rm WZ} = -\omega^2 T\det(\partial_i x^{\mu}) + T\mathcal{L}_{nr} + \mathcal{O}(\omega^{-2})\,. \tag{2.18}$$

Because of a crucial cancellation, the term that is singular as $\omega\to\infty$ is exactly the same total derivative as in the bosonic case, and may be discarded. The $\omega\to\infty$ limit may then be taken, leading to the non-relativistic action

$$S_{nr} = -T\int d^{p+1}\xi\,\sqrt{-\det g}\left[\frac{1}{2}\hat{g}^{ij}\mathbf{u}_i\cdot\mathbf{u}_j + i\bar{\theta}_{+}\hat{\gamma}^{i}\partial_i\theta_{+}\right] + T\int b_{nr}\,. \tag{2.19}$$

As we shall see shortly, this lagrangian simplifies dramatically after fixing the kappa gauge symmetry. First, we note that it is invariant under a ‘non-relativistic spacetime supersymmetry’ . If one sets

$$\epsilon = \sqrt{\omega}\,\epsilon_{-} + \frac{1}{\sqrt{\omega}}\,\epsilon_{+} \tag{2.20}$$

for (rescaled) Γ_*-eigenspinor parameters $\epsilon_{\pm}$, then the the original supersymmetry transformations (1.13) have the $\omega\to\infty$ limit

$$\delta\theta_{\pm} = \epsilon_{\pm}, \qquad \delta x^{\mu} = i\bar{\theta}_{-}\Gamma^{\mu}\epsilon_{-}\,, \qquad \delta x^{a} = 2i\bar{\theta}_{-}\Gamma^{a}\epsilon_{+}\,. \tag{2.21}$$

Note the simple transformation law of x^a, defined in (2.5).

The non-relativistic kappa-symmetry transformations are found in a similar way from the expansion

$$\Gamma_{\kappa} = \Gamma_{*} - \frac{1}{\omega}\left[\hat{\gamma}^{k}u_{k}{}^{a}\Gamma_{a}\Gamma_{*}\right] + \mathcal{O}\left(\omega^{-2}\right). \tag{2.22}$$

In terms of the Γ_* eigenspinor parameters $\kappa_{\pm}$ defined by

$$\kappa = \sqrt{\omega}\,\kappa_{-} + \frac{1}{\sqrt{\omega}}\,\kappa_{+}\,, \tag{2.23}$$

the original kappa-symmetry transformations (2.2) become, in the $\omega \to \infty$ limit,

$$\begin{aligned} \delta\theta_- &= \kappa_- \,, & \delta\theta_+ &= -\frac{1}{2}\hat{\gamma}^i u_i{}^a \Gamma_a \kappa_- \,, \\ \delta x^\mu &= -i\bar{\theta}_- \Gamma^\mu \kappa_- \,, & \delta x^a &= -2i\bar{\theta}_+ \Gamma^a \kappa_- \,. \end{aligned} \tag{2.24}$$

Note that only κ_- appears in these transformations. As a consequence, *the non-relativistic kappa transformation is an irreducible gauge symmetry*, in contrast to the infinitely reducible gauge symmetry in the relativistic case.

The non-relativistic gauge invariance (2.24) allows us to set $\theta_- = 0$, in which case

$$b_{nr} = -\frac{i}{p!}(\theta_+ \gamma_{i_1 \dots i_p} d\theta_+) d\xi^{i_1} \cdots d\xi^{i_p} \,. \tag{2.25}$$

The non-relativistic lagrangian density in this gauge is

$$\mathcal{L}_{nr} = -\sqrt{-\det g}\,\left[\frac{1}{2} g^{ij} \partial_i \mathbf{x} \cdot \partial_j \mathbf{x} + 2i\bar{\theta}_+ \gamma^i \partial_i \theta_+\right]. \tag{2.26}$$

Fixing the reparametrization invariance by the Monge gauge choice we arrive at the fully gauge-fixed lagrangian density

$$\mathcal{L} = -\frac{1}{2}\eta^{ij} \partial_i \mathbf{x} \cdot \partial_j \mathbf{x} - 2i\theta_+ \Gamma^i \partial_i \theta_+ \,. \tag{2.27}$$

Our non-relativistic limit was designed to capture the linearly realized supersymmetry as a limit of the (super)symmetries of the relativistic theory, so we should now verify that this has been accomplished. In the gauge $\theta_- = 0$, a supersymmetry transformation of the type (2.21) must be accompanied by a compensating (non-relativistic) kappa-gauge transformation. This leads to transformations for which only $\delta\theta_+$ and δx^a are non-zero:

$$\delta\theta_+ = \epsilon_+ + \frac{1}{2}\gamma^i \partial_i x^a \Gamma_a \epsilon_- , \qquad \delta x^a = 2i\bar{\theta}_+ \Gamma^a \epsilon_- \,. \tag{2.28}$$

We see that θ_+ is the Goldstone fermion for a fermionic 'shift' symmetry, with parameter ϵ_+, which is all that is left of the non-linearly realized supersymmetry of the gauge-fixed relativistic action. The parameter ϵ_- is the parameter of a linearly-realized worldvolume supersymmetry.

The condition for supersymmetry preservation is $\delta\theta_+ = 0$. As can be explicitly verified, this is the non-relativistic limit of the condition $\Gamma_\kappa \epsilon = -\epsilon$. In the relativistic case, the supersymmetry preserving condition is $\Gamma_\kappa \epsilon = \pm\epsilon$, but the non-relativistic limit forces a choice of sign.

3. The D0-brane

The super D0-brane [12] is a direct generalization of the lagrangian for the relativistic massive superparticle in four dimensions [13, 14]. In view of our discussion in the introduction of the meaning of the non-relativistic limit, we set $c = 1$. The lagrangian is

$$L = -M\left(\sqrt{-\pi^2} - i\bar{\theta}\Gamma_{11}\dot{\theta}\right), \qquad \pi^m := \dot{X}^m + i\bar{\theta}\Gamma^m \dot{\theta} \,. \tag{3.1}$$

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The action is invariant under time-reparametrizations and the κ-symmetry gauge transformations,

$$\delta\theta = \frac{1}{2}\left(1 + \frac{\pi_m \Gamma^m}{\sqrt{-\pi^2}}\Gamma_{11}\right)\kappa\,, \qquad \delta X^m = -i\bar\theta\Gamma^m\delta\theta\,. \tag{3.2}$$

In order to take the non-relativistic limit, we set

$$M = \omega\, m\,, \qquad X^0 = \omega\, x^0\,, \qquad \theta = \sqrt{\omega}\,\theta_- + \frac{1}{\sqrt{\omega}}\theta_+\,, \tag{3.3}$$

where $\theta_\pm$ are ± 1 eigenspinors of[2]

$$\Gamma_* = \Gamma^0\Gamma_{11}\,. \tag{3.4}$$

Each has 16 linearly-independent real components. We now have

$$\pi^0 = \omega\, e + \frac{i}{\omega}\,\bar\theta_+\Gamma^0\dot\theta_+\,, \qquad \pi^a = u^a\,, \tag{3.5}$$

where

$$e = \dot x^0 + i\bar\theta_-\Gamma^0\dot\theta_-\,, \qquad u^a = \dot x^a + 2i\bar\theta_+\Gamma^a\dot\theta_-\,, \qquad x^a = X^a + i\bar\theta_-\Gamma^a\theta_+, \tag{3.6}$$

and

$$-i\bar\theta\Gamma_{11}\dot\theta = -i\omega\,\bar\theta_-\Gamma^0\dot\theta_- + \frac{i}{\omega}\,\bar\theta_+\Gamma^0\dot\theta_+\,. \tag{3.7}$$

The lagrangian (3.1) can now be shown to take the form

$$L = -\omega^2 m\,\dot x^0 + \frac{m}{2e}\,|\mathbf{u}|^2 - 2im\,\bar\theta_+\Gamma^0\dot\theta_+ + \mathcal{O}\left(\frac{1}{\omega^2}\right). \tag{3.8}$$

The first term, which is singular as $\omega \to \infty$, is the same total time derivative that we discarded in the purely bosonic case, and we can do the same again here; note that this feature depends on a cancellation that occurs only for the kappa-symmetric action. The $\omega \to \infty$ limit may now be taken and we thus arrive at the time-reparametrization invariant form of the non-relativistic D0-brane lagrangian

$$L = \frac{m}{2e}\,|\mathbf{u}|^2 + 2im\,\theta_+^T\dot\theta_+\,. \tag{3.9}$$

Naturally, the action is still invariant under supersymmetry, and κ-symmetry gauge transformations, albeit in modified form. In terms of the parameters $\epsilon_\pm$ we have

$$\delta\theta_\pm = \epsilon_\pm\,, \qquad \delta x^0 = i\bar\theta_-\Gamma^0\epsilon_-\,, \qquad \delta x^a = 2i\bar\theta_-\Gamma^a\epsilon_+\,. \tag{3.10}$$

The non-relativistic kappa gauge transformations involve only κ_- and are

$$\begin{aligned} \delta\theta_- &= \kappa_-\,, & \delta\theta_+ &= -\frac{1}{2e}\,\Gamma^0 u^a\Gamma_a\kappa_-\,, \\ \delta x^0 &= -i\bar\theta_-\Gamma^0\kappa_-\,, & \delta x^a &= -2i\bar\theta_+\Gamma^a\kappa_-\,. \end{aligned} \tag{3.11}$$

[2]This differs from (1.25) for the reason stated earlier.

We may fix the kappa gauge transformation by the gauge choice $\theta_- = 0$. If we also fix the time reparametrization invariance by choosing $\dot{x}^0 = 1$, then we find the fully gauge fixed lagrangian

$$L = \frac{m}{2}|\dot{\mathbf{x}}|^2 + 2im(\theta_+)^T\dot{\theta}_+ \,, \tag{3.12}$$

which is quadratic in the 8 real 'bosons' $\mathbf{x}$ and the 16 real 'fermions' θ_+. The spacetime supersymmetry transformations of the variables of the lagrangian must be accompanied by a compensating κ-transformations in order to maintain the choice of gauge. The resulting non-zero 'physical' supersymmetry transformations are

$$\delta\theta_+ = \epsilon_+ + \frac{1}{2}\Gamma^0\dot{x}^a\Gamma_a\,\epsilon_- \,, \qquad \delta x^a = 2i\bar{\theta}_+\Gamma^a\epsilon_- \,. \tag{3.13}$$

4. Super-Bargmann algebra

We have seen how the supersymmetric action for the non-relativistic D0-brane emerges from the relativistic action in a suitable limit. Here we shall show that an analogous limit applied to the 10-dimensional $N = 2$ super-Poincaré algebra yields the super-Bargmann symmetry algebra of the non-relativistic D0-brane. We should note here that other supersymmetrizations of the Bargmann algebra have been considered in other contexts. For example [14]; in addition, super-Bargmann algebras with vector supercharges appear in [15, 16]. However, to the best of our knowledge, the particular super-Bargmann algebra that we present here has not previously appeared in the literature.

The non-zero (anti)commutation relations of the 10-dimensional $N = 2$ super-Poincaré algebra, with central charge $\mathcal{Z}$, are (supressing spinor indices)

$$\begin{aligned}
[P_m,\ M_{pq}] &= -i(\eta_{mp}P_q - \eta_{mq}P_p)\,,\\
[M_{mn},\ M_{pq}] &= -i(\eta_{np}M_{mq} - \eta_{mp}M_{nq} - \eta_{nq}M_{mp} + \eta_{mq}M_{np})\,,\\
[Q,\ M_{mn}] &= \frac{i}{2}Q\Gamma_{mn}\,,\\
\{Q,\ Q\} &= 2(C\Gamma^m)P_m + 2(C\Gamma_{11})\mathcal{Z}\,.
\end{aligned} \tag{4.1}$$

For the purpose of taking the non-relativistic limit we set

$$P_0 = -\frac{1}{2\omega}H - \omega Z\,, \qquad \mathcal{Z} = \omega Z - \frac{1}{2\omega}H, \qquad M_{a0} = \omega B_a \tag{4.2}$$

and

$$Q = \sqrt{\omega}\,Q_+ + \frac{1}{\sqrt{\omega}}\,Q_- \,, \tag{4.3}$$

where $Q_\pm$ are ± 1 eigenspinors of the matrix Γ_* of (3.4). Note that the rescaling of the components of Q is 'opposite' to the rescaling of the components of θ in (1.29), so that $Q\theta$ remains finite as $\omega \to \infty$. The super-Bargmann algebra is obtained in the $\omega \to \infty$ limit. Using the identities

$$\Gamma_{ab}\mathbb{P}_\pm = \mathbb{P}_\pm\Gamma_{ab}\,, \qquad \Gamma_{0a}\mathbb{P}_\pm = \mathbb{P}_\mp\Gamma_{0a}\,, \qquad \mathbb{P}_\pm^T C = C\mathbb{P}_\mp\,, \tag{4.4}$$

where

$$\mathbb{P}_\pm = \frac{1}{2}\left(1 \pm \Gamma_*\right), \qquad (\Gamma_* = \Gamma^0\Gamma_{11}), \tag{4.5}$$

one finds that the non-zero (anti)commutators in the $\omega \to \infty$ limit are

$$\begin{aligned}
[M_{ab}, M_{cd}] &= -i\left(\eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac} + \eta_{ad}M_{bc}\right), \\
[P_a, M_{bc}] &= -i\left(\eta_{ab}P_c - \eta_{ac}P_b\right), \\
[B_a, M_{bc}] &= -i\left(\eta_{ab}B_c - \eta_{ac}B_b\right), \\
[H, B_a] &= iP_a\,, \\
[Q_\pm, M_{ab}] &= \frac{i}{2}Q_\pm\Gamma_{ab}\,, \\
[Q_-, B_a] &= \frac{i}{2}Q_+\Gamma_{a0}\,, \\
\{Q_-, Q_-\} &= 2H\mathbb{P}_-\,, \\
\{Q_+, Q_-\} &= 2\left(C\Gamma^a\mathbb{P}_-\right)P_a\,,
\end{aligned} \tag{4.6}$$

which defines the super-Galilei algebra, and

$$[P_a, B_b] = i\delta_{ab}Z\,, \qquad \{Q_+, Q_+\} = 4Z\mathbb{P}_+\,, \tag{4.7}$$

which extends the super-Galilei algebra to the super-Bargmann algebra via the central charge Z. Note that the sub-superalgebra spanned by (H, Q_-) is a conventional $N = 16$ worldline supersymmetry algebra.

The super-Bargmann algebra is realized as a symmetry algebra of the non-relativistic D0-brane lagrangian (3.9). The even generators are

$$\begin{aligned}
H &= -p_0, \qquad P_a \;=\; p_a\,, \qquad J_{ab} = x_{[a}p_{b]} - \frac{1}{2}\zeta_+\,\Gamma_{ab}\,\theta_+ - \frac{1}{2}\zeta_-\,\Gamma_{ab}\,\theta_-\,, \\
B_a &= p_a x^0 - \frac{1}{2}\zeta_+\,\Gamma_{a0}\,\theta_- - m\left(x_a + \,i\overline{\theta}_+\Gamma_a\theta_-\right) - \frac{i}{2}\overline{\theta}_-\Gamma_{ba}\Gamma_0\theta_- p^b\,, \\
Z &= m
\end{aligned} \tag{4.8}$$

and the odd generators are

$$Q_- = -i\;\zeta_- + \overline{\theta}_-(\Gamma^0 p_0)\,, \qquad Q_+ = -i\;\zeta_+ + 2\overline{\theta}_-(\Gamma^a p_a) - 2m\;\overline{\theta}_+\Gamma^0\,. \tag{4.9}$$

Note that the central charge Z is the particle's mass. Both occurrences of it in (4.7) arise from the central charge $\mathcal{Z}$ in the original super-Poincaré algebra. The non-relativistic particle and the relativistic massive superparticle have both frequently been used to illustrate the relevance of central charges in particle mechanics (see, for example, [17]). It is thus amusing to note (in agreement with a result of [18] in a different context) that these two examples become essentially the same example in the context of the non-relativistic D0-brane!

To conclude this section, we observe that there is a one-parameter family of contractions of the Poincaré algebra, obtained by setting

$$P_0 = -\frac{1}{2\omega}H - \omega Z\,, \qquad \mathcal{Z} = k\omega Z - \frac{1}{2\omega}H\,, \qquad M_{a0} = \omega B_a \tag{4.10}$$

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for constant k. The $k = 1$ case was considered above. In general, one finds (as before) the Bargmann algebra with central charge Z in the $\omega \to \infty$ limit. For any k there is a contraction of the super-Poincaré algebra obtainable by setting

$$Q = \sqrt{\omega} S \,. \tag{4.11}$$

This leads to the direct sum of the Bargmann algebra with a 32-dimensional odd algebra defined by the anticommutator

$$\{S, S\} = 2Z(1 + k\Gamma^0\Gamma_{11}) \,. \tag{4.12}$$

This is not a supersymmetry algebra, but it *is* the superalgebra realized by the $p = 0$ analog of the lagrangian (1.27) for $k = Q/T$ (i.e., a non-BPS D0-brane for $k \neq 1$). For $|k| = 1$, *but not otherwise*, the alternative rescaling (4.3) of the fermionic generators is possible. As we have seen this alternative rescaling allows a contraction to a non-relativistic superalgebra that contains a standard worldline supersymmetry algebra as a sub-superalgebra.

5. The IIA superstring

We now turn to the IIA superstring. The action is [19]

$$S = -T \int d^2\sigma \, \left[\sqrt{-\det G} - \mathcal{L}_{\rm WZ}\right] , \tag{5.1}$$

where the WZ lagrangian density is

$$\mathcal{L}_{\rm WZ} = -\varepsilon^{jk} i\, \bar\theta \Gamma_m \Gamma_{11} \partial_j \theta \,({\Pi_k}^m - \frac{i}{2}\bar\theta\Gamma^m\partial_k\theta) \,. \tag{5.2}$$

The action is invariant under the kappa-gauge transformation

$$\delta\theta = \frac{1}{2}(1 - \Gamma_\kappa)\kappa \,, \qquad \delta X^m = -i\bar\theta\Gamma^m\delta\theta \,, \tag{5.3}$$

where

$$\Gamma_\kappa \equiv \frac{1}{2}\frac{\varepsilon^{jk}}{\sqrt{-G}}(\Pi_j^m\Gamma_m)(\Pi_k^n\Gamma_n)\Gamma_{11} \,. \tag{5.4}$$

The non-relativistic limit for strings is special in that *there is no need to rescale the string tension.* Only the dynamical variables need be rescaled, and we do this by setting

$$X^\mu = \omega\, x^\mu \,, \quad \theta = \sqrt{\omega}\, \theta_- + \frac{1}{\sqrt{\omega}}\, \theta_+ \,, \tag{5.5}$$

where $\theta_\pm$ are the ± 1 eigenspinors of the matrix

$$\Gamma_* = \Gamma_0\Gamma_1\Gamma_{11} \,. \tag{5.6}$$

We now have

$${\Pi_i}^\mu = \omega\, {e_i}^\mu + \frac{i}{\omega}\, \bar\theta_+\Gamma^\mu\partial_i\theta_+ \,, \qquad {\Pi_i}^a = {u_i}^a \,, \tag{5.7}$$

where

$$e_i{}^\mu = \partial_i x^\mu + i\bar{\theta}_-\Gamma^\mu\partial_i\theta_- \,, \tag{5.8}$$

and

$$u_j{}^b = \partial_j x^b + 2i\bar{\theta}_+\Gamma^b\partial_j\theta_- \,, \qquad \left(x^a = X^a + i\bar{\theta}_-\Gamma^a\theta_+\right). \tag{5.9}$$

Proceeding as before, we find that the terms in the action proportional to ω^2 combine to form a term proportional to $\varepsilon^{jk}\varepsilon_{\mu\nu}(\partial_j x^\mu \partial_k x^\nu)$. This is a total derivative, which we may drop. Taking the $\omega \to \infty$ limit then yields

$$\mathcal{L} = -\frac{1}{2}e\,\hat{g}^{jk}\,\mathbf{u}_j\cdot\mathbf{u}_k \,-\, 2i\,e\,(\bar{\theta}_+\hat{\gamma}^i\partial_i\theta_+) - 2i\,\varepsilon^{jk}\left(\bar{\theta}_+\Gamma_a\Gamma_{11}\,\partial_j\theta_-\right)\left(u_k{}^a - i\,\bar{\theta}_+\Gamma^a\,\partial_k\theta_-\right). \tag{5.10}$$

The spacetime supersymmetry transformations are obtained in the way explained earlier, with the result that

$$\delta\theta_\pm = \epsilon_\pm \,, \qquad \delta x^\mu = i\bar{\theta}_-\Gamma^\mu\epsilon_- \,, \qquad \delta x^a = 2i\bar{\theta}_-\Gamma^\mu\epsilon_+ \,. \tag{5.11}$$

Using the invariance of the 1-forms e^μ, $\mathbf{u}$ and $d\theta$, and the standard cyclic identity of the 10-dimensional Dirac matrices, one can show that the variation of the lagrangian (5.10) is a total derivative.

The non-relativistic kappa gauge transformation is also found as before, with the result that

$$\begin{aligned} \delta\theta_- &= \kappa_- \,, & \delta\theta_+ &= -\frac{1}{2}\hat{\gamma}^i u_i{}^a\Gamma_a\kappa_- \\ \delta x^\mu &= -i\bar{\theta}_-\Gamma^\mu\kappa_- \,, & \delta x^a &= -2i\bar{\theta}_+\Gamma^a\kappa_- \,. \end{aligned} \tag{5.12}$$

This gauge invariance allows us to choose $\theta_- = 0$. If we further fix the reparametrization invariance by the usual Monge gauge then we arrive at the non-relativistic IIA superstring action

$$S = -T\int d^2\xi\left[\frac{1}{2}\eta^{ij}\partial_i\mathbf{x}\cdot\partial_j\mathbf{x} \,+\, 2i\,\bar{\theta}_+\Gamma^i\partial_i\theta_+\right]. \tag{5.13}$$

The supersymmetry transformations of the gauge-fixed variables appearing in this action are obtained by combining the supersymmetry transformation with a compensating kappa-transformation, with the result that

$$\delta\theta_+ = \epsilon_+ \,+\, \frac{1}{2}\gamma^i\partial_i x^a\Gamma_a\epsilon_- \,, \qquad \delta x^a = 2i\bar{\theta}_+\Gamma^a\epsilon_- \,. \tag{5.14}$$

6. Conclusions and comments

In this paper we have shown (extending earlier work [1]–[4]) that there is a natural p-brane generalization of the non-relativistic limit of the action for a point particle in which the limiting gauge-fixed action is a worldvolume Poincaré invariant field theory. The main result of this paper is a generalization of this non-relativistic limit to superbranes. We have shown that there is a limit of the reparametrization invariant and kappa-gauge invariant super-p-brane (made possible by a crucial cancellation between the Dirac and WZ lagrangians

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of terms which are separately singular in the limit) with the feature that the spacetime supersymmetry of the gauge-invariant action implies a worldvolume supersymmetry of the gauge-fixed action, exactly as happens for relativistic superbranes. An interesting feature of this limit is that the non-relativistic kappa-gauge invariance is irreducible, in contrast to the reducibility of the relativistic kappa-gauge invariance. Our results apply to all cases in which the supersymmetric worldvolume field theory involves only scalars and spinors, and we have given explicit lagrangians for $p = 0, 1, 2$, including the gauge-invariant lagrangian of the non-relativistic M2-brane. We expect that a similar result will apply to super-D-p-branes and the M5-brane.

We have discussed the case of the D0-brane in detail. In particular, we have shown that our non-relativistic limit corresponds to a particular contraction of the super-Poincaré symmetry algebra of the relativistic D0-brane. The contracted algebra is a supersymmetric extension of the Bargmann algebra, which is indeed realized by the non-relativistic D0-brane, and which contains a conventional $N = 16$ worldline supersymmetry algebra as a sub-super-algebra. One can anticipate that similar results will apply for $p > 0$, thus extending to superbranes the non-relativistic brane algebras introduced in [10].

We have also discussed in detail the case of the IIA superstring, obtaining the reparametrization invariant and kappa-symmetric action for a non-relativistic IIA superstring. We were motivated to consider this case separately because explicit results in this case could be useful for a supersymmetric extension of the non-relativistic string theory of [2, 3].

Our treatment of the non-relativistic limit of p-branes for general p brings to light a special feature of the $p = 1$ case. Recall that the limit generally involves a rescaling of the p-brane tension. For example, for $p = 0$ the limit involves a rescaling in which the ratio $\omega = M/m$ of the relativistic particle mass M to the non-relativistic particle mass m is taken to infinity. In other words, for fixed m one takes $M \to \infty$, as expected (given that we have fixed the speed of light) because this corresponds to an effectively infinite rest-mass energy that makes particle-antiparticle pair production impossible. Curiously, for $p = 2$ one has the inverse relation $\omega = T_{nr}/T_{\rm rel}$ for non-relativistic and relativistic tensions, so that the non-relativistic limit corresponds to taking $T_{\rm rel}$ to zero for fixed T_{nr} (and fixed speed of light). This can be understood by viewing the relativistic p-brane action as a $(p + 1)$-dimensional field theory with coupling constant $g \sim 1/\sqrt{T_p}$. For $p \geq 1$ the non-relativistic limit is an ultra-violet (UV) limit because we focus on a small patch of the brane and then rescale so that the patch becomes the entire brane. For $p \geq 2$ the coupling constant g is dimensionful and is driven to infinity in the UV limit, whereas for $p = 1$ the coupling constant is dimensionless and no (classical) scaling is needed.

Finally, we should emphasise that we have concentrated in this paper on the non-relativistic limit of superbranes in a Minkowski vacuum. There are further interesting issues in relation to non-flat backgrounds that we hope to address in future work. For example, the ideas developed here may have implications for the 'extra-supersymmetry' puzzle arising in the field theory limit of the M2-brane in certain hyper-Kähler backgrounds [20].

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References

[1] P.K. Townsend, *Brane theory solitons*, hep-th/0004039.

[2] J. Gomis and H. Ooguri, *Non-relativistic closed string theory*, *J. Math. Phys.* **42** (2001) 3127 [hep-th/0009181].

[3] U.H. Danielsson, A. Guijosa and M. Kruczenski, *IIA/B, wound and wrapped*, *J. High Energy Phys.* **10** (2000) 020 [hep-th/0009182].

[4] J.A. Garcia, A. Guijosa and J.D. Vergara, *A membrane action for OM theory*, *Nucl. Phys.* **B 630** (2002) 178 [hep-th/0201140].

[5] M.J. Duff, *Comment on time-variation of fundamental constants*, hep-th/0208093.

[6] E. Bergshoeff, E. Sezgin and P.K. Townsend, *Supermembranes and eleven-dimensional supergravity*, *Phys. Lett.* **B 189** (1987) 75.

[7] J. Hughes and J. Polchinski, *Partially broken global supersymmetry and the superstring*, *Nucl. Phys.* **B 278** (1986) 147.

[8] A. Achúcarro, J.P. Gauntlett, K. Itoh and P.K. Townsend, *World volume supersymmetry from space-time supersymmetry of the four-dimensional supermembrane*, *Nucl. Phys.* **B 314** (1989) 129.

[9] A. Achúcarro, J.M. Evans, P.K. Townsend and D.L. Wiltshire, *Super p-branes*, *Phys. Lett.* **B 198** (1987) 441.

[10] J. Brugues, T. Curtright, J. Gomis and L. Mezincescu, *Non-relativistic strings and branes as non-linear realizations of Galilei groups*, *Phys. Lett.* **B 594** (2004) 227 [hep-th/0404175].

[11] J. Hughes, J. Liu and J. Polchinski, *Supermembranes*, *Phys. Lett.* **B 180** (1986) 370.

[12] E. Bergshoeff and P.K. Townsend, *Super D-branes*, *Nucl. Phys.* **B 490** (1997) 145 [hep-th/9611173].

[13] R. Casalbuoni, *The classical mechanics for Bose-Fermi systems*, *Nuovo Cim.* **A33** (1976) 389.

[14] J.A. de Azcárraga and J. Lukierski, *Supersymmetric particles with internal symmetries and central charges*, *Phys. Lett.* **B 113** (1982) 170; *Supersymmetric particles in $N = 2$ superspace: phase space variables and hamiltonian dynamics*, *Phys. Rev.* **D 28** (1983) 1337.

[15] J.P. Gauntlett, J. Gomis and P.K. Townsend, *Supersymmetry and the physical phase space formulation of spinning particles*, *Phys. Lett.* **B 248** (1990) 288.

[16] C. Duval and P.A. Horvathy, *On Schrödinger superalgebras*, *J. Math. Phys.* **35** (1994) 2516.

[17] J.P. Gauntlett, J. Gomis and P.K. Townsend, *Particle actions as Wess-Zumino terms for space-time (super)symmetry groups*, *Phys. Lett.* **B 249** (1990) 255.

[18] J.A. de Azcárraga and D. Ginestar, *Nonrelativistic limit of supersymmetric theories*, *J. Math. Phys.* **32** (1991) 3500.

[19] M.B. Green and J.H. Schwarz, *Covariant description of superstrings*, *Phys. Lett.* **B 136** (1984) 367.

[20] P.K. Townsend, *The story of M*, `hep-th/0205309`.